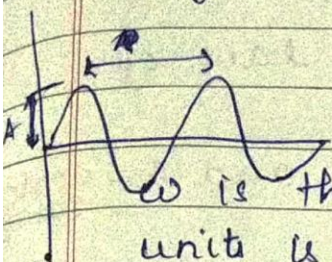
**Amplitude:** The maximum displacement A. is known as Amplitude. The period T, is the time for one revolution, since the ~~amplitude~~ angular path for one revolution is $2\pi$ rad.




$$T = \frac{2\pi \, rad}{\omega} \quad -(1)$$

$\omega$ is the angular ~~momentum~~ velocity, its SI units is $rad\,s^{-1}$. The frequency is the reciprocal of the period

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi \, rad} \quad -(2)$$

$$\omega = (2\pi \, rad)\nu$$

The mathemetical form of displacement ~~y~~ y is shown as

$$y = A \sin(\omega t) \quad -(3)$$

where $\omega t$ is known as phase.

On first differentiation

$$\frac{dy}{dt} = \omega A \cos \omega t \quad -(4)$$

On second diff. $\frac{d^2y}{dt^2} = -A\omega^2 \sin \omega t \quad -(5)$

hence $\boxed{\frac{\partial^2 y}{dt^2} = -\omega^2 y} \quad -(6)$

This equation is known as simple harmonic motion equation.

Also according to Hooke's law,

$$F = -k_n y \quad — (7)$$

where F is restoring force of a spring and y a displacement, ~~&~~ $k_n$ is force constant.

According to Newton's second law of motion.

$F = m \cdot a$ (m = mass and a = acceleration)

$$\therefore F = m\frac{d^2y}{dt^2} \quad — (8)$$

On equating eq. (6) and (8)

$$\frac{\partial^2 y}{dt^2} = \frac{F}{m} = -\omega^2 y$$

or $$-\frac{k_n y}{m} = -\omega^2 y \quad — (9)$$

or $$\omega = \sqrt{\frac{k_n}{m}}$$

$$\Rightarrow \nu = \frac{1}{2\pi}\sqrt{\frac{k_n}{m}} \quad — (10)$$

This frequency is referred as frequency of simple harmonic motion.

# The Basics of quantum mechanics

→ Schrodinger theory often referred as wave mechanics was based on the realization of wave properties of elementary particles such as electrons.

→ Using the compton effect it was deduced that

$$\lambda = \frac{h}{p} = \frac{h}{mu}$$

Since we know that electrons have wave properties

$$2\pi r = n\lambda$$

$$\text{or} \quad 2\pi r = \frac{nh}{mu}$$

$$\text{or} \quad mur = \frac{nh}{2\pi}$$

Here mur is the orbital angular momentum which according to Bohr is an integral number of $h/2\pi$.

**The uncertainity principle:-**

It states that if we avoid the momentum change using radiation of long wavelength then we won't be able to determine the change in position precisely.

Mathemetically,

uncertainity in position & → uncertainity in momentum.

$$\Delta q \cdot \Delta p \approx \frac{h}{4\pi} \approx \frac{1}{2}\hbar \qquad \left\{ \hbar = \frac{h}{2\pi} \right\}$$

Thus the product of these uncertainity decreases as the mass increases.

For time factor

$$\Delta t = \frac{\Delta q}{u}$$

Since energy is $\frac{1}{2}mu^2$ and momentum is $mu$, the uncertainity in energy is given by

$$\Delta E = u \Delta p$$

Hence the eq. becomes

$$\Delta E \cdot \Delta t \approx \frac{h}{4\pi} \approx \frac{1}{2}\hbar$$

## Schrodinger's wave mechanics

For a wave travelling in vacuum with speed c, the wave equation is

$$\frac{\partial^2 y}{dt^2} = \frac{1}{c^2}\frac{\partial^2 y}{dt^2} \quad \text{— (1)}$$

where y is displacement in y-direction

Schrodinger replaced this equation with:-

$$\frac{\partial^2 \psi}{dx^2} = \frac{1}{c^2}\frac{\partial^2 y}{dt^2} \quad \text{— (2)}$$

$\psi(x,t)$ is the function that describes the behaviour of electrons and can be written as

$$\psi(x,t) = Ce^{i\alpha}$$

where $i = \sqrt{-1}$

and $\alpha = 2\pi\left(\frac{x}{\lambda} - \nu t\right)$

$\psi(x,t)$ is known as eigen function or wave function and can be split into

$$\psi(x,t) = \cancel{Ce^{2\pi i \ldots}},\ Ce^{2\pi i x/\lambda}\, e^{-2\pi i \nu t}$$

where $\psi(x)$ is a function of x, not t

and $e^{-2\pi i \nu t}$ is function of t, not x.

On replacing $\nu$ by $E/h$ and $\lambda$ by $h/p$ we get

$$\psi(x,t) = C e^{2\pi i x p_x/h} e^{-2\pi i E t/h}$$

On partial differentiation with respect to $t$ we get.

$$\frac{\partial \psi(x,t)}{\partial t} = -\frac{2\pi i E C}{h} e^{2\pi i x p_x/h} e^{-2\pi i E t/h}$$

$$= -\frac{2\pi i E}{h} \psi(x,t)$$

On rearranging the eqn.

$$\boxed{-\frac{h}{2\pi i} \frac{\partial \psi}{\partial t} = E\psi}$$

This is an operator equation and operator $\hat{H}$ is written as

$$\frac{-h}{2\pi i} \frac{\partial}{\partial t}$$

For differentiating the same eqn with respect to $x$ we get

$$\frac{\partial \psi(x,t)}{\partial x} = \frac{2\pi i p_x}{h} \psi(x,t)$$

Rearanged to

$$\frac{h}{2\pi i} \frac{\partial \psi}{\partial x} = p_x \psi \quad \Rightarrow \hat{p}_x = \frac{h}{2\pi i} \frac{\partial}{\partial x}$$

$\hat{p}_x$ is operator for momentum

For total energy of a system

$$E_{total} = E_{kinetic} + E_{potential}$$

$$= \frac{p_x^2}{2m} + E_{pE}(x,t)$$

$$\downarrow$$

$$E_{pot}$$

On expressing E in term of momentum it is called Hamiltonian and the eqn becomes.

$$\boxed{\hat{H} = \frac{-h^2}{8\pi^2 m}\frac{\partial^2}{\partial x^2} + E_p(x,t)}$$

and overall equation is

$$\left[-\frac{h^2}{8\pi^2 m}\frac{\partial^2}{\partial x^2} + E_p(x,t)\right]\psi = \frac{-h}{2\pi i}\frac{\partial \psi}{\partial t}$$

The operator $\partial^2/\partial x^2$ can be replaced by

$$\boxed{\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}$$

$\nabla^2$ is the del-squared operator or Laplacian operator.

Hence the equation becomes

$$\left[-\frac{h^2}{8\pi^2 m}\nabla^2 + E_p(x,y,z)\right]\psi = \frac{-h}{2\pi i}\frac{\partial \psi}{\partial t}$$

Time-dependent Schrodinger wave equation.